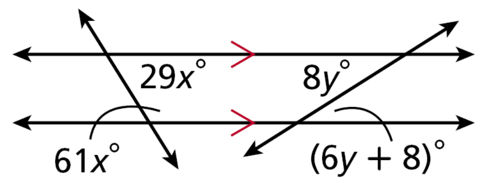
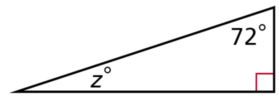
Attendance Problems. **Find the value of each variable.**

1. x **2.** y



3. z



- I can prove and apply properties of parallelograms.
- I can use properties of parallelograms to solve problems.

Common Core CC.9-12.G.CO.11 Prove theorems about parallelograms.

Any polygon with four sides is a quadrilateral. However, some quadrilaterals have special properties. These *special quadrilaterals* are given their own names.

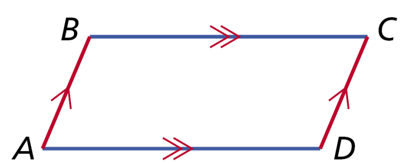
Helpful Hint

Opposite sides of a quadrilateral do not share a vertex. Opposite angles do not share a side.

A quadrilateral with two pairs of parallel sides is a **parallelogram**. To write the name of a parallelogram, you use the symbol ▱.

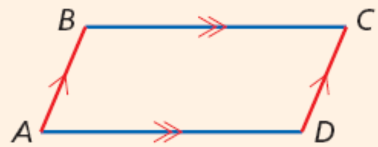
Refer to example 1 on page 404.

Parallelogram *ABCD*
▱*ABCD*

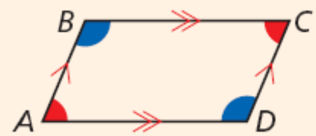
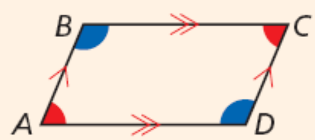
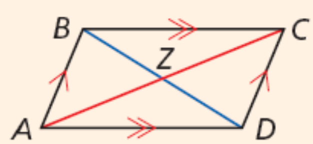


$\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$

Theorem 6-2-1 Properties of Parallelograms

THEOREM	HYPOTHESIS	CONCLUSION
If a quadrilateral is a parallelogram, then its opposite sides are congruent. (▱ → opp. sides ≅)		$\overline{AB} \cong \overline{CD}$ $\overline{BC} \cong \overline{DA}$

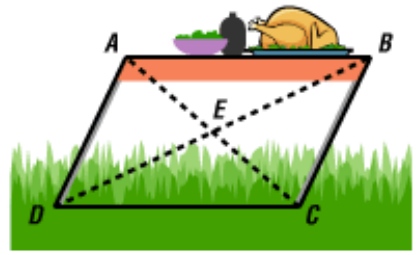
Theorems Properties of Parallelograms

	THEOREM	HYPOTHESIS	CONCLUSION
6-2-2	If a quadrilateral is a parallelogram, then its opposite angles are congruent. (▱ → opp. ∠s ≅)		$\angle A \cong \angle C$ $\angle B \cong \angle D$
6-2-3	If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. (▱ → cons. ∠s supp.)		$m\angle A + m\angle B = 180°$ $m\angle B + m\angle C = 180°$ $m\angle C + m\angle D = 180°$ $m\angle D + m\angle A = 180°$
6-2-4	If a quadrilateral is a parallelogram, then its diagonals bisect each other. (▱ → diags. bisect each other)		$\overline{AZ} \cong \overline{CZ}$ $\overline{BZ} \cong \overline{DZ}$

Question: What do you call an urgent message sent across a parallel network?
Answer: A parallelogram.

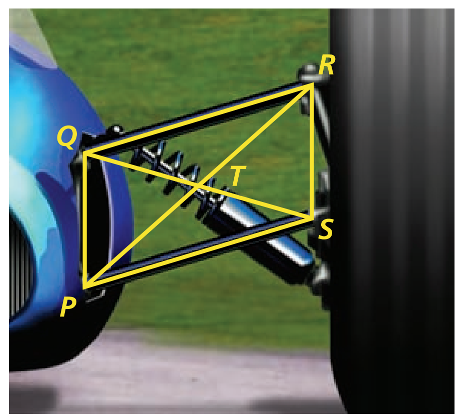
"'Tis true, 'tis pity, and pity 'tis 'tis true." -- *Poloniouius*, in Willie the Shake's Hamlet, Prince of Darkness_

<u>Video Example 1</u>. An inexpensive picnic table begins to lean over and forms a parallelogram. The diagram shows the picnic table. In $\square ABCD$, AD = 60 inches, AE = 35 inches, and $m\measuredangle CDA = 81°$. Find each measure.



1 *Racing Application*

The diagram shows the parallelogram-shaped linkage that joins the frame of a race car to one wheel of the car. In $\square PQRS$, $QR = 48$ cm, $RT = 30$ cm, and $m\angle QPS = 73°$. Find each measure.



A ***PS***

$\overline{PS} \cong \overline{QR}$	*$\square$ → opp. sides ≅*
$PS = QR$	*Def. of ≅ segs.*
$PS = 48$ cm	*Substitute 48 for QR.*

B **m∠*PQR***

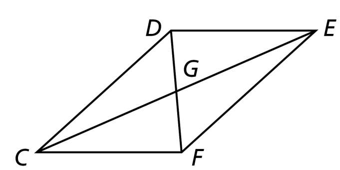
$m\angle PQR + m\angle QPS = 180°$	*$\square$ → cons. ∠s supp.*
$m\angle PQR + 73 = 180$	*Substitute 73 for m∠QPS.*
$m\angle PQR = 107°$	*Subtract 73 from both sides.*

C ***PT***

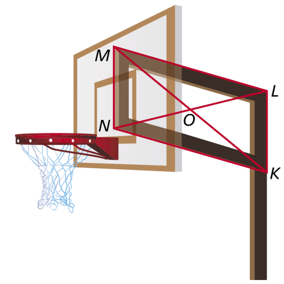
$\overline{PT} \cong \overline{RT}$	*$\square$ → diags. bisect each other*
$PT = RT$	*Def. of ≅ segs.*
$PT = 30$ cm	*Substitute 30 for RT.*

<u>Example 1</u>. In $\square CDEF$, *DE* = 74 mm, *DG* = 31 mm, and m<*FCD* = 42°. Find each measure.

A. CF **B.** $m\measuredangle EFC$ **C.** DF



<u>Guided Practice</u>. In ▱KLMN, LM = 28 in., LN = 26 in., and m<LKN = 74°. Find each measure.

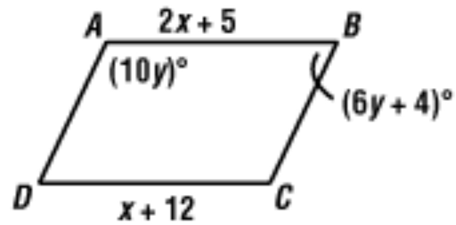


4. KN

5. $m\measuredangle NML$

6. LO

<u>**Video Example 2**</u>. Figure ABCD is a parallelogram. Find each measure.

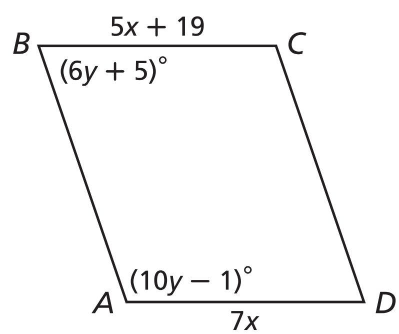


A. AB

B. $m\measuredangle B$

2 Using Properties of Parallelograms to Find Measures

***ABCD* is a parallelogram. Find each measure.**



A AD

$\overline{AD} \cong \overline{BC}$ — *▱ → opp. sides ≅*

$AD = BC$ — *Def. of ≅ segs.*

$7x = 5x + 19$ — *Substitute the given values.*

$2x = 19$ — *Subtract 5x from both sides.*

$x = 9.5$ — *Divide both sides by 2.*

$AD = 7x = 7(9.5) = 66.5$

B $m\angle B$

$m\angle A + m\angle B = 180°$ — *▱ → cons. ∠s supp.*

$(10y - 1) + (6y + 5) = 180$ — *Substitute the given values.*

$16y + 4 = 180$ — *Combine like terms.*

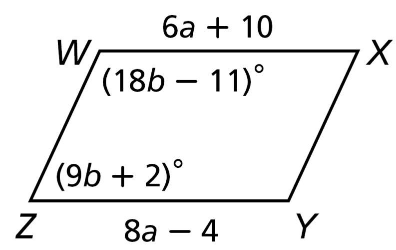
$16y = 176$ — *Subtract 4 from both sides.*

$y = 11$ — *Divide both sides by 16.*

$m\angle B = (6y + 5)° = [6(11) + 5]° = 71°$

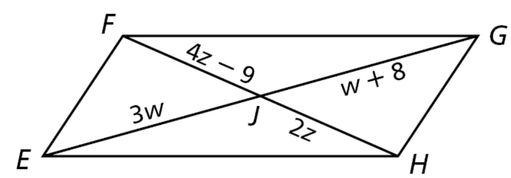
<u>Example 2</u>. *WXYZ* is a parallelogram.

A. YZ **B.** $m\measuredangle Z$



<u>Guided Practice</u>. *EFGH* is a parallelogram. Find each measure.

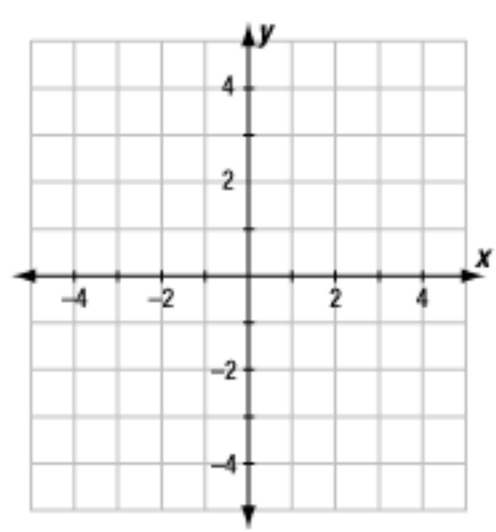
7. JG **8.** FH



Remember!

When you are drawing a figure in the coordinate plane, the name *ABCD* gives the order of the vertices.

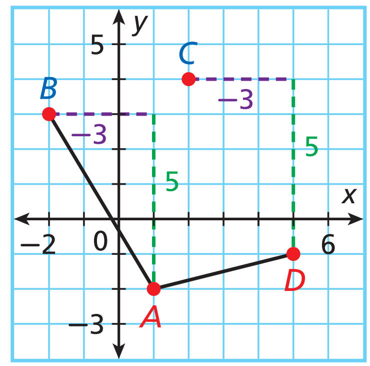
<u>Video Example 3</u>. Three vertices of a parallelogram are A(-3, 3), B(-4, 0), and C(2, -4). Find the coordinates of vertex D.



3 Parallelograms in the Coordinate Plane

Three vertices of ▱*ABCD* are $A(1, -2)$, $B(-2, 3)$, and $D(5, -1)$. Find the coordinates of vertex *C*.

Since *ABCD* is a parallelogram, both pairs of opposite sides must be parallel.



Step 1 Graph the given points.

Step 2 Find the slope of $\overline{AB}$ by counting the units from *A* to *B*.
The rise from −2 to 3 is 5.
The run from 1 to −2 is −3.

Step 3 Start at *D* and count the same number of units.
A rise of 5 from −1 is 4.
A run of −3 from 5 is 2. Label $(2, 4)$ as vertex *C*.

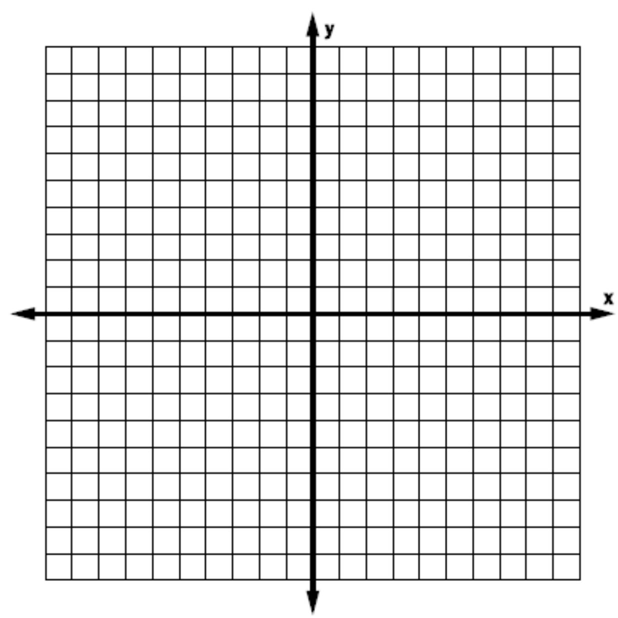
Step 4 Use the slope formula to verify that $\overline{BC} \parallel \overline{AD}$.

$$\text{slope of } \overline{BC} = \frac{4-3}{2-(-2)} = \frac{1}{4}$$

$$\text{slope of } \overline{AD} = \frac{-1-(-2)}{5-1} = \frac{1}{4}$$

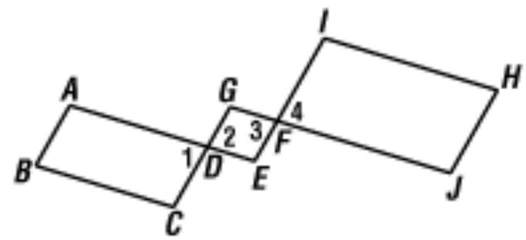
The coordinates of vertex *C* are $(2, 4)$.

9. <u>Guided Practice</u>. Three vertices of ▱ *PQRS* are *P*(–3, –2), *Q*(–1, 4), and *S*(5, 0). Find the coordinates of vertex *R*.



6-2 Properties of Parallelograms
(p 407) 17-25 odd.

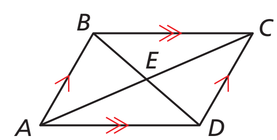
<u>Video Example 4.</u>
Given: ABCD, DEFG, & FIHJ are parallelograms. A, D, and E are collinear. C, D, and G are collinear. J, F, and G are collinear and E, F, and I are collinear.



4 Using Properties of Parallelograms in a Proof

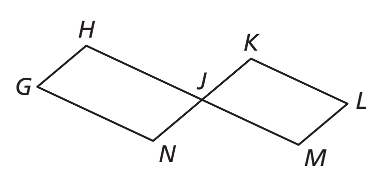
Write a two-column proof.

A Theorem 6-2-2
Given: *ABCD* is a parallelogram.
Prove: $\angle BAD \cong \angle DCB$, $\angle ABC \cong \angle CDA$
Proof:



Statements	Reasons
1. *ABCD* is a parallelogram.	1. Given
2. $\overline{AB} \cong \overline{CD}$, $\overline{DA} \cong \overline{BC}$	2. ▱ → opp. sides $\cong$
3. $\overline{BD} \cong \overline{BD}$	3. Reflex. Prop. of $\cong$
4. $\triangle BAD \cong \triangle DCB$	4. SSS *Steps 2, 3*
5. $\angle BAD \cong \angle DCB$	5. CPCTC
6. $\overline{AC} \cong \overline{AC}$	6. Reflex. Prop. of $\cong$
7. $\triangle ABC \cong \triangle CDA$	7. SSS *Steps 2, 6*
8. $\angle ABC \cong \angle CDA$	8. CPCTC

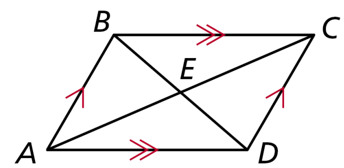
B Given: *GHJN* and *JKLM* are parallelograms. *H* and *M* are collinear. *N* and *K* are collinear.
Prove: $\angle G \cong \angle L$
Proof:



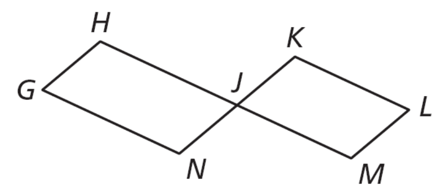
Statements	Reasons
1. *GHJN* and *JKLM* are parallelograms.	1. Given
2. $\angle HJN \cong \angle G$, $\angle MJK \cong \angle L$	2. ▱ → opp. $\angle$s $\cong$
3. $\angle HJN \cong \angle MJK$	3. Vert. $\angle$s Thm.
4. $\angle G \cong \angle L$	4. Trans. Prop. of $\cong$

Example 4.

A. Given: ABCD is a parallelogram.
Prove: $\triangle AEB \cong \triangle CED$

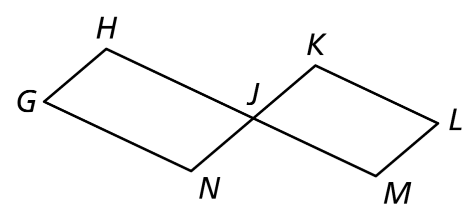


B. Given: *GHJN* and *JKLM* are parallelograms. *H* and *M* are collinear. *N* and *K* are collinear.
Prove: $\measuredangle H \cong \measuredangle M$



10. Guided Practice.

Given: Given: *GHJN* and *JKLM* are parallelograms. *H* and *M* are collinear. *N* and *K* are collinear.
Prove: $\measuredangle N \cong \measuredangle K$



6-2 Properties of Parallelograms *(p 407)* 17-25 odd, 26, 28, 42, 44, 48-50.